### **ADVANCED MATH CONCPETS**

POWER OBJECTIVE #1  Perform arithmetic operations with complex numbers and use complex numbers in polynomial identities and equations.  (N.CN.)  Polynomials with real coefficients  N.CN.1 Know there is a complex number is such that i = -1, and every complex number has the form a + bi with a and b real.  N.CN.2 Use the relation i = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.  N.CN.3 (+)Find the conjugate of a complex number; use conjugates to find moduli and quadretis of complex numbers.  N.CN.4 (+)Extend polynomial identities to the complex numbers. For example, rewrite x' + 4 as (x + 2)(x + 2)).  N.CN.9 (+)Extend polynomial identities to the complex numbers. For example, rewrite x' + 4 as (x + 2)(x + 2)).  N.CN.9 (+)Extend polynomials.  CONCEPTUAL CATEGORY  DOMAIN  Seeing Structures in Expressions  A.SSE  Polynomial and rational  SUPPORTING INDICATORS  A.SSE.1 interpret the structure of expressions in equivalent forms to solve problems. (A.SSE)  Polynomial and rational  SUPPORTING INDICATORS  A.SSE.1. Interpret expressions that represent a quantity in terms of its context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1. Interpret parts of an expression by viewing one or more of their parts as a single entity. For example, interpret P(1+r)* as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x* - y* as (x* y* - y* - x* - y* -	CONCEPTUAL CATEGORY	NUMBER AND QUANTITY (N)	
POWER OBJECTIVE #1  Perform arithmetic operations with complex numbers and use complex numbers in polynomial identities and equations. (N.CN)  Polynomials with real coefficients  N.CN.1 Know there is a complex number is such that i' = -1, and every complex number has the form a bi with a ond breal.  N.CN.2 Use the relation i' = -1 and the communative, associative, and distributive properties to add, subtract, and multiply complex numbers.  N.CN.3 (+)Entend the conjugate of a complex number; use conjugates to find moduli and quatients of complex numbers.  N.CN.5 (*)Extend polynomial identities to the complex numbers. For example, rewrite x' + 4 as (x + 2)(x - 2).  N.CN.9 (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.  CONCEPTUAL CATEGORY  DOMAIN  Seeing Structures in Expressions  ALGEBRA (A)  DOMAIN  Seeing Structures in Expressions in equivalent forms to solve problems. (A.SSE)  Polynomial and rational  SUPPORTING INDICATORS  A.SSE.1 Interpret expressions that represent a quantity in terms of its context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret Party as the product of P and a factor not depending on P.  A.SSE.1 b Interpret complicated expression to identify ways to rewrite it. For example, see x' - y' as (x' y'') c' y''), thus recognizing it as a difference of squares that can be factored as (x' - y')(x' - y').  A.SSE.1 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	DOMAIN	The Complex Number System N.CN	Grading Period
Complex number has the form a + bi with a and b real.	POWER OBJECTIVE #1	complex numbers in polynomial identities and equations. (N.CN)  Polynomials with real coefficients	
N.CN.3 (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.  N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.  N.CN.8 (+)Extend polynomial identities to the complex numbers. For example, rewrite x² + 4 as (x + 2)[x - 2].  N.CN.9 (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.  CONCEPTUAL CATEGORY  DOMAIN  Seeing Structures in Expressions  A.SSE  Write and interpret the structure of expressions in equivalent forms to solve problems. (A.SSE)  Polynomial and rational  SUPPORTING INDICATORS  A.SSE.1 Interpret expressions that represent a quantity in terms of its context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)" as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x² - y¹ as (x²)² - (y²)², thus recognizing it as a difference of squares that can be factored as (x² - y²) (x² + y²).  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	SUPPORTING INDICATORS	complex number has the form a + bi with a and b real.	
Solutions.   N.CN.8   (+)Extend polynomial identities to the complex numbers. For example, rewrite x² + 4 as (x + 2i)(x - 2i).   N.CN.9   (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.		<b>N.CN.3</b> (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	
N.CN.9 (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.    CONCEPTUAL CATEGORY		solutions.  N.CN.8 (+)Extend polynomial identities to the complex numbers. For	
POWER OBJECTIVE #2  Write and interpret the structure of expressions in equivalent forms to solve problems. (A.SSE)  Polynomial and rational  SUPPORTING INDICATORS  A.SSE.1 Interpret expressions that represent a quantity in terms of its context.  A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) <sup>n</sup> as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x <sup>d</sup> - y <sup>d</sup> as (x <sup>2</sup> ) <sup>2</sup> - (y <sup>2</sup> ) <sup>2</sup> , thus recognizing it as a difference of squares that can be factored as (x <sup>2</sup> - y <sup>2</sup> )(x <sup>2</sup> + y <sup>2</sup> ).  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.  **DOMAIN**  Arithmetic with Polynomials & Rational Expressions  A.APR  POWER OBJECTIVE #3  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)		N.CN.9 (+)Know the Fundamental Theorem of Algebra; show that it is true	
POWER OBJECTIVE #2  Write and interpret the structure of expressions in equivalent forms to solve problems. (A.SSE)  Polynomial and rational  A.SSE.1 Interpret expressions that represent a quantity in terms of its context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)" as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x <sup>4</sup> - y <sup>4</sup> as (x <sup>2</sup> ) <sup>2</sup> - (y <sup>2</sup> ) <sup>2</sup> , thus recognizing it as a difference of squares that can be factored as (x <sup>2</sup> - y <sup>2</sup> )(x <sup>2</sup> + y <sup>2</sup> ).  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	CONCEPTUAL CATEGORY	ALGEBRA (A)	
SUPPORTING INDICATORS	DOMAIN	Seeing Structures in Expressions A.SSE	Grading Period
context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.  A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) <sup>n</sup> as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x <sup>4</sup> - y <sup>4</sup> as (x <sup>2</sup> ) <sup>2</sup> - (y <sup>2</sup> ) <sup>2</sup> , thus recognizing it as a difference of squares that can be factored as (x <sup>2</sup> - y <sup>2</sup> )(x <sup>2</sup> + y <sup>2</sup> ).  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  A.APR  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	POWER OBJECTIVE #2	forms to solve problems. (A.SSE)	
A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) <sup>n</sup> as the product of P and a factor not depending on P.  A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x <sup>4</sup> - y <sup>4</sup> as (x <sup>2</sup> ) <sup>2</sup> - (y <sup>2</sup> ) <sup>2</sup> , thus recognizing it as a difference of squares that can be factored as (x <sup>2</sup> - y <sup>2</sup> )(x <sup>2</sup> + y <sup>2</sup> ).  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  A.APR  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	SUPPORTING INDICATORS	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.  A.SSE.1.a Interpret parts of an expression, such as terms, factors, and	
example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .  A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational  Expressions  A.APR  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)		<b>A.SSE.1.b</b> Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of	
the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *  DOMAIN  Arithmetic with Polynomials & Rational Expressions  A.APR  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)		example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .	
Expressions  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)		the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *	
POWER OBJECTIVE #3  Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use polynomials identities to solve polynomials. (A.APR)	DOMAIN	•	Grading Period
Beyond quadratic  SUPPORTING INDICATORS  A.APR.1 Understand that polynomials form a system analogous to the	POWER OBJECTIVE #3	Perform arithmetic operations on polynomials, understand the relationship between zeros and factors of polynomials and use	

	graphically. Use this understanding as a process of reasoning and explain the reasoning. (A.REI)  Combine polynomial, rational, radical, absolute value, and exponential functions  Simple radical and rational	
POWER OBJECTIVE #6	Understand, represent and solve equations and inequalities	Period
DOMAIN	same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.  Reasoning with Equations & Inequalities A.REI	Grading
	nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.  A.CED.4 Rearrange formulas to highlight a quantity of interest, using the	
	<b>A.CED.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or populable entions in a modeling context. For example, represent inequalities	
	<b>A.CED.2</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
SUPPORTING INDICATORS	<b>A.CED.1</b> Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	
POWER OBJECTIVE #5	(A.CED)  Equations using all available types of expressions, including simple root functions	
DOMAIN	Creating Equations A.CED	Grading Period
	<b>A.APR.7</b> (+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	<b>.</b>
SUPPORTING INDICATORS	<b>A.APR.6</b> Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $a(x) + \frac{r(x)}{b(x)}$ , where $a(x)$ , $b(x)$ , $a(x)$ , and $a(x)$ are polynomials with the degree of $a(x)$ less than the degree of $a(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.	
SUPPORTING INDICATORS	Linear and quadratic denominators	
POWER OBJECTIVE #4	Rewrite rational expressions. (A.APR)	
	<b>A.APR.5</b> (+)Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. <sup>1</sup>	
	<b>A.APR.4</b> Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	
	<b>A.APR.3</b> Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	and a number a, the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .	
	integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. <b>A.APR.2</b> Know and apply the Remainder Theorem: For a polynomial p(x)	

SUPPORTING INDICATORS	A.REI.2 Solve simple rational and radical equations in one variable, and	
SOFFORTING INDICATORS	give examples showing how extraneous solutions may arise.	
	<b>A.REI.11</b> Explain why the x-coordinates of the points where the graphs of	
	the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation	
	f(x) = g(x); find the solutions approximately, e.g., using technology to graph	
	the functions, make tables of values, or find successive approximations.	
	Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute	
	value, exponential, and logarithmic functions.*	
CONCEPTUAL CATEGORY	Functions (F)	L
DOMAIN	Interpreting Functions F.IF	Grading Period
POWER OBJECTIVE #7	Interpret functions that arise in applications in terms of the	renou
	context and analyze functions using different representations.	
	(F.IF)	
	Emphasize selection of appropriate models	
	Focus on using key features to guide selection of appropriate type of model	
SUPPORTING INDICATORS	function  F.IF.4 For a function that models a relationship between two quantities,	
SUPPORTING INDICATORS	interpret key features of graphs and tables in terms of the quantities, and	
	sketch graphs showing key features given a verbal description of the	
	relationship. Key features include: intercepts; intervals where the function is	
	increasing, decreasing, positive, or negative; relative maximums and	
	minimums; symmetries; end behavior; and periodicity.*	
	<b>F.IF.5</b> Relate the domain of a function to its graph and, where applicable, to	
	the quantitative relationship it describes. For example, if the function $h(n)$	
	gives the number of person-hours it takes to assemble n engines in a factory,	
	then the positive integers would be an appropriate domain for the function. *	
	F.IF.6 Calculate and interpret the average rate of change of a function	
	(presented symbolically or as a table) over a specified interval. Estimate the	
	rate of change from a graph.*	
	F.IF.7 Graph functions expressed symbolically and show key features of the	
	graph, by hand in simple cases and using technology for more complicated	
	cases.*	
	F.IF.7.b Graph square root, cube root, and piecewise-defined functions,	
	including step functions and absolute value function.	
	F.IF.7.c Graph polynomial functions, identifying zeros when suitable	
	factorizations are available, and showing end behavior.	
	F.IF.7.d (+)Graph rational functions, identifying zeros and asymptotes when	
	suitable factorizations are available, and showing end behavior.	
	<b>F.IF.7.e</b> Graph exponential and logarithmic functions, showing intercepts	
	and end behavior, and trigonometric functions, showing period, midline, and	
	amplitude.	
	<b>F.IF.8</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
	F.IF.8 a. Use the process of factoring and completing the square in a	
	quadratic function to show zeros, extreme values, and symmetry of	
	the graph, and interpret these in terms of a context.	
	<i>F.IF.8</i> b. Use the properties of exponents to interpret expressions for	
	exponential functions. For example, identify percent rate of change in	
	functions such as $y = (1.02)t$ , $y = (0.97)t$ , $y = (1.01)12t$ , $y = (1.2)t/10$ ,	

	and classify them as representing exponential growth or decay.	
	<b>F.IF.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	
DOMAIN	Building Functions F.BF	Grading Period
POWER OBJECTIVE #8	Build a function that models a relationship between two quantities and build new functions from existing functions.  (F.BF)  Include all types of functions studied Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types	
SUPPORTING INDICATORS	<ul> <li>F.BF.1 Write a function that describes a relationship between two quantities.*</li> <li>F.BF.1.b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</li> <li>F.BF.1.c (+)Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.</li> <li>F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</li> <li>F.BF.4. Find inverse functions.</li> <li>F.BF.4.a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2 x³ or f(x) = (x+1)/(x-1) for x ≠ 1.</li> <li>F.BF.4.b (+)Verify by composition that one function is the inverse of another.</li> <li>F.BF.4.c (+)Read values of an inverse function from a graph or a table, given that the function has an inverse.</li> <li>F.BF.5 (+)Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</li> </ul>	
DOMAIN	Linear, Quadratic & Exponential Models F.LE	Grading Period
POWER OBJECTIVE #9	Construct and compare linear, quadratic, and exponential models and solve problems. (F.LE)  Logarithms as solutions for exponentials	renou
SUPPORTING INDICATOR	<ul> <li>F.LE 1. Distinguish between situations that can be modeled with linear functions and with exponential functions</li> <li>F.LE b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</li> <li>F.LE c. Recognize situations in which a quantity grows or decays by a</li> </ul>	

	constant percent rate per unit interval relative to another.	
	F.LE. 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	<b>F.LE.4</b> For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.	
	<b>F.LE.5</b> Interpret the parameters in a linear or exponential function in terms of a context	
DOMAIN	Trigonometric Functions F.TF	Grading Period
POWER OBJECTIVE #10	Extend the domain of trigonometric functions using the unit circle, model periodic phenomena with trigonometric functions and prove and apply trigonometric identities. (F.TF)	
SUPPORTING INDICATORS	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	<b>F.TF.3</b> (+)Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $x$ , $x$ + $x$ , and $x$ - $x$ in terms of their values for $x$ , where $x$ is any real number.	
	<ul> <li>F.TF.4 (+)Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</li> <li>F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*</li> </ul>	
	F.TF.7 (+)Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*	
	<b>F.TF.8</b> Prove the Pythagorean identity $\sin^2(\vartheta) + \cos^2(\vartheta) = 1$ and use it to find $\sin(\vartheta)$ , $\cos(\vartheta)$ , or $\tan(\vartheta)$ given $\sin(\vartheta)$ , $\cos(\vartheta)$ , or $\tan(\vartheta)$ and the quadrant of the angle.	
CONCEPTUAL CATEGORY	GEOMETRY (G)	
DOMAIN	<b>Expressing Geometric Properties with Equations</b>	
	G.GPE	
POWER OBJECTIVE #11	Translate between the geometric description and the equation for a conic section. (G.GPE)	
	<b>G.GPE.</b> 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
SUPPORTING INDICATORS	G.GPE.2. Derive the equation of a parabola given a focus and directrix.  G.GPE.3 (+)Derive the equations of ellipses and hyperbolas given the foci,	
SOFFORTING INDICATORS	using the fact that the sum or difference of distances from the foci is constant.	